1.5 Infinite Geometric Series

Focus On ...

• generalizing a rule for determining the sum of an infinite geometric series
• explaining why a geometric series is convergent or divergent
• solving a problem that involves a geometric sequence or series

Infinite Geometric Series

Example: \( \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots \)

Idea of Partial Sum

add the first two terms, the first three terms, and so on.

\[ S_1 = \frac{1}{2} \]
\[ S_2 = \]
\[ S_3 = \]
\[ S_4 = \]

Are the partial sums approaching a particular value?
Distinguish between a convergent and divergent series

<table>
<thead>
<tr>
<th>Geometric Series</th>
<th>Ratio (r)</th>
<th>Partial Sum</th>
<th>Convergent/ Divergent</th>
</tr>
</thead>
<tbody>
<tr>
<td>2+4+8+16....</td>
<td></td>
<td></td>
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<tr>
<td>(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots)</td>
<td></td>
<td></td>
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<tr>
<td>-1+1+1+1+...</td>
<td></td>
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<tr>
<td>1+1+1+1+...</td>
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</tbody>
</table>

Check to see if the partial sums approach a particular value as the number of terms get larger.

If the partial sums approach a particular value as the number of terms get larger, the series is convergent. If the partial sums do not approach a particular value the series is divergent.

Question:

In the above examples, why is it the value of \(r\) cannot equal 0?
Convergent Geometric Series:

- a series with an infinite number of terms, in which the sequence of partial sums approaches a fixed value
- occurs when \(-1 < r < 1\)

Example: Consider the series: \(\frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \frac{3}{16} + \ldots\)

\[
S_1 = \\
S_2 = \\
S_3 = \\
S_4 = 
\]

As the number of terms increases, the sum of the series approaches ____?

Divergent Geometric Series:

- a series with an infinite number of terms, in which the sequence of partial sums does not approach a fixed number
- occurs when \(r > 1\) or \(r < -1\)

Example: Consider the series: \(4 + 8 + 16 + 32 + \ldots\)

\[
S_1 = 4 \\
S_2 = 12 \\
S_3 = 28 \\
S_4 = 60 \\
S_5 = 124
\]

As the number of terms increases, the sum of the series continues to grow.
INVESTIGATION: p. 58

1. Start with a square piece of paper.
   a) Draw a line dividing it in half.
   b) Shade one of the halves.
   c) In the unshaded half of the square, draw a line to divide it in half. Shade one of the halves.
   d) Repeat part c) at least six more times.

2. Write a sequence of terms indicating the area of each newly shaded region as a fraction of the entire page. List the first five terms.

3. Predict the next two terms for the sequence.

4. Is the sequence arithmetic, geometric, or neither? Justify your answer.

5. Write a sequence of shaded regions. Write the rule for the $n^{th}$ term of the sequence.

6. Ignoring physical limitations, could this sequence continue? In other words, would this be an infinite sequence? Explain your answer.

7. What can you conclude about the total of all areas added together?
Infinite Geometric Series:

In the formula, \[ S_n = \frac{t_1(1-r^n)}{1-r} \]

as \( n \) gets very large, the term \( r^n \to 0 \) when \(-1 < r < 1\)

Therefore the formula becomes:

\[ S_\infty = \frac{t_1}{1-r}, \text{ where } -1 < r < 1 \]

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Example 1: Sum of an Infinite Geometric Series

Decide whether each infinite geometric series is convergent or divergent. State the sum of the series, if it exists.

a) \( 1 - \frac{1}{3} + \frac{1}{9} - \ldots \)  
   b) \( 2 - 4 + 8 - \ldots \)
Example 2: Your Turn

Determine whether each infinite geometric series converges or diverges. Calculate the sum, if it exists.

a) \(1 + \frac{1}{5} + \frac{1}{25} + \ldots\)  
b) \(4 + 8 + 16 + \ldots\)

Example 3: Apply the Sum of an Infinite Geometric Series

Assume that each shaded square represents \(\frac{1}{4}\) of the area of the larger square bordering two of its adjacent sides and that the shading continues indefinitely in the indicated manner.

a) Write the series of terms that would represent this situation.

b) How much of the total area of the largest square is shaded?
Section 1.5 Infinite Geometric Series

Example 4: Your Turn

You can express $0.584$ as an infinite geometric series.

\[ 0.584 = 0.584 \ 584 \ 584 \ldots = 0.584 + 0.000 \ 584 + 0.000 \ 000 \ 584 + \ldots \]

Determine the sum of the series.

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Key Ideas

- An infinite geometric series is a geometric series that has an infinite number of terms; that is, the series has no last term.

- An infinite series is said to be convergent if its sequence of partial sums approaches a finite number. This number is the sum of the infinite series. An infinite series that is not convergent is said to be divergent.

- An infinite geometric series has a sum when $-1 < r < 1$ and the sum is given by

\[ S_n = \frac{t_1}{1 - r}. \]

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SeatWork/Homework

p.63-65

#1b, 2abd, 3a, 4, 5ac, 6-9, 12, 13, 15, 17, 22, 23 (handout)